A trail & error method is sufficient to balance a simple chemical equation, but the following algebraic method may be used to solve "complicated" chemical equations.

For demonstration purposes, to balance the following (simple) chemical equation

$$a\ C_2H_6 + b\ O_2 \rightarrow c\ CO_2 + d\ H_2O$$

use the concept of the conservation of mass, the meaning of the subscripts in the chemical formula, and the coefficients, to produce the following system of equations:

for carbon: $2a = c$

for hydrogen: $6a = 2d$

for oxygen: $2b = 2c + d.$

As there are 4 variables and 3 equations, the system of equations can't be "solved" (i.e. there won't be an unique solution), so

let $d = 1$

now, the preceding system of equations become

$$\begin{aligned} 2a - c &= 0 \\ 6a &= 2 \\ 2b - 2c &= 1 \end{aligned}$$

which may be solved using matrix algebra[1]. Matrix algebra may be viewed as simply a method to express a system of equations in a less intimidating manner; thereby simplifying its solution. The above system of equations may be expressed as an augmented matrix

$$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 6 & 0 & 0 & 2 \\ 0 & 2 & -2 & 1 \end{array}\right]$$

To manipulate the matrix, use various matrix operations:

step 1. divide the second row by 6 and the third row by 2

$$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & -1 & \frac{1}{2} \end{array}\right]$$

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[1] the Gauss-Jordan elimination (1, 2) is the simplest method to solve a system of equations, which is used on this page and in some calculators (3a, 3b (youtube)). Alternatively, use Cramer's rule (4, 5), or an inverse matrix (6, 7), which could be implemented in MS Excel (8).

step 2. multiply the second row by 2, then subtract it from the first row

$$\left[\begin{array}{ccc|c} 0 & 0 & -1 & -\frac{2}{3} \\ 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & -1 & \frac{1}{2} \end{array}\right]$$

step 3. multiply the first row by – 1

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & \frac{2}{3} \\ 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & -1 & \frac{1}{2} \end{array}\right]$$

step 4. add the first row to the third row, then move the first row to the third row

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{7}{6} \\ 0 & 0 & 1 & \frac{2}{3} \end{array}\right]$$

thus,

$a = \frac{1}{3}$

$b = \frac{7}{6}$

$c = \frac{2}{3}$

$d = 1$

hence,

$$\frac{1}{3}\,C_2H_6 + \frac{7}{6}\,O_2 \rightarrow \frac{2}{3}\,CO_2 + H_2O$$

alternatively, multiply all coefficients by 6

$$2\ C_2H_6 + 7\ O_2 \rightarrow 4\,CO_2 + 6\,H_2O.$$